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Quantum-like information processing using vector solitons

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Abstract

The concept of simulating the quantum logic via collisions of vector solitons (Janutka 2006 *J. Phys. A: Math. Gen.* **39** 12505, 2007 *J. Phys. A: Math. Theor.* **40** 10813) is developed in the direction of designing a true quantum-information processor that is based on mesoscopic objects, solitons. In this concept, quantum-like information is encoded in the vector-soliton (polarization) parameters. An exponential increase of the logical-operating speed compared to that achievable in the earlier simulation schemes is found to be possible due to a coherent conversion of a 2^n-component vector soliton that carries an n-cebit of information into an ensemble of 2^{n-1} two-component and 2^{n-2} four-component vector pulses. Two solid-state circuits (transmitting magnetic solitons or fluxons of long Josephson junctions) which enable such a pulse conversion are proposed.

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1. Introduction

The implementation of quantum-information processing using any microscopic object (the spin or charge of a quantum particle) as a qubit is an extremely difficult task because of the fast decoherence and the complexity of the quantum state readout. In order to avoid these difficulties, an effort has been made to propose alternative classical systems carrying out logical operations on cebits (classical qubit alternatives whose state is determined by a vector of complex components) [1][1]. These efforts have been only partially successful since they enabled simulating the quantum algorithms, however, with an exponential increase of resources necessary to simulate any logical operation (an exponential growth of state-vector switches) with the number of cebits. Thus, the main goal of quantum-computer science; the

[1] We note that the cebit definition of [1] is wider than that of [2] since it allows multi-cebit state entanglement.

reduction of the number of elementary operations leading to solutions of computational tasks compared to those used in classical algorithms has not been achieved. The source of the failure has been determined in [1] to be the lack of nonlocality of the information encoded in the state vector of the classical systems. The nonlocality of the information-register state can be thought of as a property relevant to systems built of the qubits encoded in separate particles, while the state of the single-particle spin is not nonlocal although both the information registers can be relevant to the same state vector. However, there is no proof that the nonlocality is a solely quantum property and one can hope for building a classical system (of cebits and their switchers) performing quantum-logical algorithms.

The aim of the present paper is to propose such an alternative way to implement schemes of quantum-information processing using mesoscopic objects (vector solitons). Such objects maintain coherence for a relatively long time and their state parameters are classical observables, thus their states are not destroyed in the act of measurement. Furthermore, the continuous character of the classical-system decoherence is different to that of the quantum ones. It does not lead to an instant damage of the state vector. The present proposal is based on a (recently introduced by the author) concept of simulation of the quantum logic which utilizes the possibility of performing any unitary operation via colliding the vector solitons [3, 4]. In this concept, the memory-register state is represented by a normalized 2^n-component vector of complex numbers (the soliton-polarization vector) similar to the n-qubit state vector. The elementary logical (gate) operations consist of the sequences of collisions of the register soliton with other vector pulses of at most two non-zero components. Since the information encoded in this vector is not nonlocal, the number of soliton collisions necessary to simulate any logical operation increases exponentially with the number of cebits (it is of the order of the number of polarization components). The present study shows that the processing with the nonlocal information utilizing soliton collisions can be performed via dynamically disjoining the many-component soliton into simpler two-component vector solitons (of the Manakov type) and then carrying out (one-cebit) logical-gate operations on them independently but simultaneously. After performing the gate operation, the pulses have to connect to the 2^n-component vector soliton. The result is the exponential reduction of the time consumed by the gate operation compared to the time of the sequential gate realization. In order to perform the two-cebit (*CNOT*) operations, the register pulse has to be divided into the four-component vector solitons.

Two different circuit realizations are studied in the present paper. The first one is a pulse-transmission line of 2^n connected parallel ferromagnetic wires. More precisely, we consider a long ferromagnetic plate whose cross-section thickness changes periodically along the direction perpendicular to the pulse-transmission direction and each of the thick areas of the plate (a path) transmits a magnetic vortex being a component of a vector magnetic soliton. On some length of the plate, the pulse-component-transmitting paths are disconnected (there are slits in the plate). Entering the slit area, the 2^n-component vector soliton divides into a number of less complex (two- or four-component) vector solitons. The second circuit proposal is a pulse-transmission line of 2^n parallel long Josephson junctions joined via a common superconducting plate. The vector soliton (fluxon) whose components relate to the consecutive long Josephson junctions can be divided into less complex vector fluxons when there are slits in the (common for different junctions) superconducting plate.

Section 2 contains the outline of the idea of the quantum-logical operating via vector-soliton collisions. Schemes of elementary quantum-gate (one- and two-cebit) operations are demonstrated. In section 3, the coherent conversion of a vector soliton into a number of pulses of a lower polarization dimension is analyzed with relevance to both the studied soliton (ferromagnetic and Josephson-junction) transmission lines. In section 4, the method of the fast gate-operation performing via the soliton-ensemble collisions is described. Section 5 is devoted to the discussion of difficulties of the soliton-circuit building and to a short outline of previous investigations of complex (ferromagnetic and Josephson-junction) soliton-transmission lines.

2. Quantum-logic simulation via soliton collisions

I describe the idea of performing elementary quantum-logical operations via vector-soliton collisions [3] which is the basis of further investigations of the present paper.

2.1. Basics of vector solitons

The vector solitons (first analyzed by Manakov in the context of the self-focusing optical beams in nonlinear media including two light-polarization directions [5]) are characterized by a number of parameters some of which change during the collision with other vector pulses. In particular, the one-soliton solutions of the $2N$-component vector nonlinear Schrödinger equation

$$\mathrm{i}\frac{\partial\epsilon_j}{\partial\tau}+\frac{\partial^2\epsilon_j}{\partial x^2}+\frac{1}{2}\sum_{k=1}^{2N}|\epsilon_k|^2\epsilon_j=0 \tag{1}$$

takes the form

$$\epsilon_j(x,\tau)=4\mathrm{i}c_j\zeta''\exp[\mathrm{i}2\zeta'x+\mathrm{i}4(\zeta'^2-\zeta''^2)\tau]\,\mathrm{sech}[2\zeta''(x-x_0)+8\zeta'\zeta''\tau]. \tag{2}$$

Here c_j denote components of a (complex) polarization vector of the unit length $\boldsymbol{c}$, while the constant ζ ($\zeta'\equiv\mathrm{Re}\,\zeta$, $\zeta''\equiv\mathrm{Im}\,\zeta$) represents a (complex) wave number. Two colliding solitons change their polarizations without changing their velocities and pulse-envelope shapes (the wave numbers remain unchanged). During the collision with the soliton of the parameters ζ_y', ζ_y'', ($\zeta_y\equiv\zeta_y'+\mathrm{i}\zeta_y''$), x_{0y}, $\boldsymbol{c}_y$, the polarization vector $\boldsymbol{c}$ of (2) transforms into

$$\boldsymbol{c}'=\frac{1}{\chi}\frac{\zeta^*-\zeta_y}{\zeta^*-\zeta_y^*}\left[\boldsymbol{c}+\frac{\zeta_y-\zeta_y^*}{\zeta_y^*-\zeta^*}(\boldsymbol{c}_y^*\cdot\boldsymbol{c})\boldsymbol{c}_y\right], \tag{3}$$

where

$$\chi\equiv\chi(\boldsymbol{c},\boldsymbol{c}_y)=\frac{|\zeta-\zeta_y^*|}{|\zeta-\zeta_y|}\left[1+\frac{(\zeta-\zeta^*)(\zeta_y^*-\zeta_y)}{|\zeta-\zeta_y|^2}|\boldsymbol{c}^*\cdot\boldsymbol{c}_y|^2\right]^{1/2}. \tag{4}$$

This result, found using the inverse scattering transform [5], can also be achieved with a direct (Hirota) method of solving (1) [6]. Also, soliton solutions of the equations of the optical multi-component self-induced transparency are known to be characterized by similar parameters ζ', ζ'', x_0, $\boldsymbol{c}$ and their polarizations $\boldsymbol{c}$ transform during the collisions following (3) [7, 8].

2.2. Elementary logical operations

The 2^n-components of the soliton polarization $\boldsymbol{c}=(c_1,\ldots,c_{2^n})$ can be identified with the coefficients of the n-cebit state. In particular, for $n=3$,

$$\begin{aligned}|\boldsymbol{c}\rangle&=c_1|0\rangle|0\rangle|0\rangle+c_2|0\rangle|0\rangle|1\rangle+c_3|0\rangle|1\rangle|0\rangle+c_4|0\rangle|1\rangle|1\rangle\\&\quad+c_5|1\rangle|0\rangle|0\rangle+c_6|1\rangle|0\rangle|1\rangle+c_7|1\rangle|1\rangle|0\rangle+c_8|1\rangle|1\rangle|1\rangle.\end{aligned} \tag{5}$$

Following (3), the collision with a switching soliton transforms it into $\boldsymbol{c}' = \frac{1}{\chi(\boldsymbol{c}_y,\boldsymbol{c})}\mathcal{L}(\boldsymbol{c}_y)\boldsymbol{c}$, where

$$\mathcal{L}_{ij}(\boldsymbol{c}_y) = \begin{cases} \dfrac{\zeta^* - \zeta_y}{\zeta^* - \zeta_y^*}\left(1 - \dfrac{\zeta_y - \zeta_y^*}{\zeta^* - \zeta_y^*} c_{yi} c_{yj}^*\right) & i = j \\ -\dfrac{\zeta^* - \zeta_y}{\zeta^* - \zeta_y^*}\dfrac{\zeta_y - \zeta_y^*}{\zeta^* - \zeta_y^*} c_{yi} c_{yj}^* & i \neq j, \end{cases} \tag{6}$$

and $\chi(\boldsymbol{c}_y, \boldsymbol{c}) = |\mathcal{L}(\boldsymbol{c}_y)\boldsymbol{c}|$. One can perform the one-cebit and two-cebit gate operations of the universal set: *CNOT*, *Hadamard*, $\pi/8$, *phase* [9], colliding the information-register soliton with the pulses of $\boldsymbol{c}_y$, ζ_y such that $\mathcal{L}(\boldsymbol{c}_y)$ is unitary (then $\chi(\boldsymbol{c}_y, \boldsymbol{c}) = 1$).

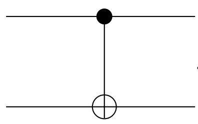
Let us look at the consecutive-gate realizations relevant to the operating with the two-cebit (*CNOT*) and one-cebit (*Hadamard*, $\pi/8$, *phase*) information registers.

2.2.1. CNOT gate. We assume the parameters of the (four-component) register pulse and switching pulse to satisfy the conditions $\zeta'' \ll \zeta_y''$ together with $|\zeta' - \zeta_y'| \ll \zeta_y''$. Taking the components of the polarization vector of the switching soliton $c_{yi} \equiv |c_{yi}|\,\mathrm{e}^{\mathrm{i}\varphi_{yi}}$ such that

$$c_{y1} = c_{y2} = 0, \qquad |c_{y3}| = |c_{y4}| = \tfrac{1}{\sqrt{2}}, \qquad \varphi_{y3} - \varphi_{y4} = (2k+1)\pi, \tag{7}$$

where k is an integer, one finds

$$\mathcal{L}(\boldsymbol{c}_y) \approx (-1)\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} \tag{8}$$

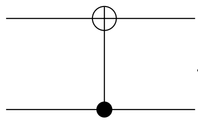


Up to the multiplier '-1', $\mathcal{L}(\boldsymbol{c}_y)$ is the *CNOT* operator represented graphically by the quantum circuit above. Exchanging the control and switched cebits, the *CNOT* operation is performed with

$$c_{y1} = c_{y3} = 0, \qquad |c_{y2}| = |c_{y4}| = \tfrac{1}{\sqrt{2}}, \qquad \varphi_{y2} - \varphi_{y4} = (2k+1)\pi \tag{9}$$

and the relevant transformation matrix of the register polarization is

$$\mathcal{L}(\boldsymbol{c}_y) \approx (-1)\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \tag{10}$$



2.2.2. Hadamard gate. Let the parameters of the (two-component) register pulse and switching pulse satisfy the same conditions as these of *CNOT* gate: $\zeta'' \ll \zeta_y''$, $|\zeta' - \zeta_y'| \ll \zeta_y''$. The polarization-vector components of the switching soliton satisfy

$$|c_{y2}| = \sqrt{\frac{1+\sqrt{2}}{2\sqrt{2}}}, \qquad \varphi_{y1} - \varphi_{y2} = (2k+1)\pi, \tag{11}$$

then

$$\mathcal{L}(\boldsymbol{c}_y) \approx \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \tag{12}$$

which is the *Hadamard* transform.

2.2.3. $\pi/8$ and phase gates. We perform the $\pi/8$ operation with the switching pulse polarized in one of the main directions, $|c_{y2}| = 1$, and of the wave number satisfying

$$\frac{\zeta^* - \zeta_y}{\zeta^* - \zeta_y^*} = \mathrm{e}^{\mathrm{i}\pi/4}. \tag{13}$$

Under these conditions, the collision transforms the cebit following the $\pi/8$ operation composed with the multiplication of the state vector by 'i',

$$\mathcal{L}(\boldsymbol{c}_y) = \mathrm{i}\begin{pmatrix} 1 & 0 \\ 0 & \mathrm{e}^{\mathrm{i}\pi/4} \end{pmatrix}. \tag{14}$$

Let us note that the *phase* operation is the composition of two $\pi/8$ gates.

2.2.4. State-vector multiplication by a number. The multiplication of the $2N$-component polarization $\boldsymbol{c}$ by a factor $\mathrm{e}^{\mathrm{i}\varphi}$ is performed with $2N$ collisions of the register soliton and switching solitons polarized along the different main axes. The wave numbers ζ_y of these switching pulses have to satisfy

$$\frac{\zeta^* - \zeta_y}{\zeta^* - \zeta_y^*} = \mathrm{e}^{\mathrm{i}\varphi/5}. \tag{15}$$

The multiplication of the state vector by '-1' and by '$-$i' has to be done as the second step of performing the *CNOT* and $\pi/8$ operations, respectively.

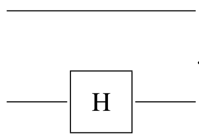
2.3. Logical operating with many-cebit information

For simplicity, let us consider in detail the action of a one-cebit gate on the two-cebit (four-component) register soliton. For instance, let us see how the *Hadamard* operation is performed on any of the two cebits. We assemble two collisions of the register soliton with switching solitons of wave numbers ζ_y, η_y and of polarizations $\boldsymbol{c}_y$, $\boldsymbol{d}_y$ respectively, under the conditions $\zeta'' \ll \zeta_y''$, η_y'', $|\zeta' - \zeta_y'| \ll \zeta_y''$, $|\zeta' - \eta_y'| \ll \eta_y''$. For the polarization vectors of the components ($c_{yj} = |c_{yj}|\,\mathrm{e}^{\mathrm{i}\varphi_{yj}}$, $d_{yj} = |d_{yj}|\,\mathrm{e}^{\mathrm{i}\phi_{yj}}$) given by

$$\begin{aligned} &c_{y1} = c_{y2} = d_{y3} = d_{y4} = 0, && |c_{y4}| = |d_{y2}| = \sqrt{\frac{1+\sqrt{2}}{2\sqrt{2}}}, \\ &\varphi_{y3} - \varphi_{y4} = (2k+1)\pi, && \phi_{y1} - \phi_{y2} = (2l+1)\pi, \end{aligned} \tag{16}$$

(k, l denote integers), the relevant transformation matrix and the graphical representation of the logical circuit take the form

$$\mathcal{L}(\boldsymbol{d}_y)\mathcal{L}(\boldsymbol{c}_y) \approx \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & -1 \end{pmatrix} \tag{17}$$



Using the switching solitons of

$$\begin{aligned} &c_{y1} = c_{y3} = d_{y2} = d_{y4} = 0, && |c_{y4}| = |d_{y3}| = \sqrt{\frac{1+\sqrt{2}}{2\sqrt{2}}}, \\ &\varphi_{y2} - \varphi_{y4} = (2k+1)\pi, && \phi_{y1} - \phi_{y3} = (2l+1)\pi, \end{aligned} \tag{18}$$

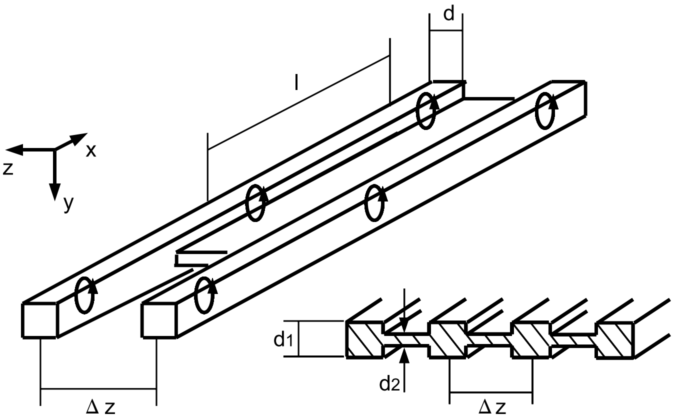
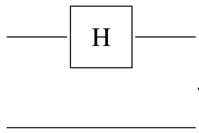


Figure 1. The double ferromagnetic transmission line for the conversion of pairs of the scalar magnetic solitons into vector solitons. Inset of the figure shows the cross section of the transmission line (a ferromagnetic plate) of the four-component solitons.

one finds the state transformation and the logical circuit

$$\mathcal{L}(\boldsymbol{d}_y)\mathcal{L}(\boldsymbol{c}_y) \approx \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \end{pmatrix} \tag{19}$$



We have seen that performing the *Hadamard* operation on any of the two cebits demands two collisions of the register soliton.

In general, performing any one-cebit gate operation on an n-cebit state demands use of 2^{n-1}-times more collisions than for a one-cebit state. Also the number of collisions necessary to perform two-cebit (*CNOT*) operation scales exponentially with n [3].

3. Dynamical change of soliton-polarization dimension

In order to perform an elementary gate operation instantly (in such a way that its time does not scales with the number of cebits), we need a method of reversible conversion of the vector pulses of a high polarization-vector dimension into a number of pulses of lower polarization dimension. It should be possible without changing the pulse velocity, the intensities and mutual phase differences of its components. For simplicity, we analyze in detail joining two scalar pulses into the two-component vector soliton; however, the present method can be applied to the many-component solitons.

3.1. Circuit of ferromagnetic wires

As the first proposal of the soliton transmission line for performing logical operations, we consider the long ferromagnetic plate of a specific shape (enabling transmission of two-component solitons) incoming two disconnected parallel ferromagnetic wires (figure 1). The Manakov-like magnetic soliton characterized by the two-component polarization vector decays into two scalar solitons when passing through the wire area.

We describe the simultaneous propagation of solitons (along the x-axis) in the two parallel wires separated (along the z-axis) by a distance Δz and magnetically ordered in the z-direction.

Following Corones [10], the envelope of the magnetic soliton in each wire is described by the nonlinear Schrödinger equation resulting from the equation of motion of the magnetic-moment density in the Heisenberg ferromagnet (the Landau–Lifshitz equation)

$$\frac{\partial \boldsymbol{m}}{\partial t} = \gamma \boldsymbol{m} \times \nabla^2 \boldsymbol{m}. \tag{20}$$

Transforming two components of the dynamical parameter into $m_+ \equiv m_x + \mathrm{i} m_y$, one finds

$$\mathrm{i}\frac{\partial m_+}{\partial t} = \gamma (m_+ \nabla^2 m_z - m_z \nabla^2 m_+). \tag{21}$$

The Holstein-Primakoff transformation

$$\begin{aligned} m_+ &= (2M)^{1/2}(1 - a^* a/2M)^{1/2} a, \\ m_z &= M - a^* a, \end{aligned} \tag{22}$$

together with the slowly varying envelope approximation ($\nabla(a^*a) \approx 0$, $\partial(a^*a)/\partial t \approx 0$) lead to the following envelope equation of motion along the x-axis ($y = z = 0$)

$$\mathrm{i}\frac{\partial \tilde{a}}{\partial \tilde{t}} + \frac{\partial^2 \tilde{a}}{\partial \tilde{x}^2} + \gamma k^2 \tilde{a}^* \tilde{a}^2 = 0. \tag{23}$$

Here $\tilde{t} = t$, $\tilde{x} = x/\sqrt{\gamma M} - 2\sqrt{\gamma M} k_x t$,

$$a(x, z, t) = \tilde{a}(x, t) \cos(k_z z)\, \mathrm{e}^{\mathrm{i}(k_x x - \gamma M k^2 t)}, \tag{24}$$

$\boldsymbol{k} = (k_x, 0, k_z)$ (we neglect the dependence of the dynamical parameter $\boldsymbol{m}$ on the y-coordinate while including its z dependence is needed for estimation of the strength of the inter-wire soliton interactions given in appendix B).

Analyzing the simultaneous soliton motion in the two parallel disconnected wires, we index the dynamical variables relevant to the magnetic excitations in both the wires by $j = 1, 2$. ($j = 1$ corresponds to a wire centered along the x-axis, $j = 2$ corresponds to a wire centered along the straight of $y = 0, z = \Delta z$.) The soliton solutions of (23) take the form

$$\tilde{a}_j = \frac{\sqrt{2}\zeta_j''}{\sqrt{\gamma} k} \mathrm{e}^{\mathrm{i}[\zeta' \tilde{x} + (\zeta_j''^2 - \zeta'^2)\tilde{t} + \varphi_j]} \operatorname{sech}[\zeta_j''(\tilde{x} - \tilde{x}_0 - 2\zeta' \tilde{t})], \tag{25}$$

(below, we call $\zeta_j \equiv \zeta' + \mathrm{i}\zeta_j''$ the wave numbers). Designing the circuit, one has to minimize an inter-wire (magnetostatic) interaction of solitons which can be attractive (stabilizing the coherent motion of the pulse pair) or repulsive (destabilizing it) depending on the phase difference $\varphi_2 - \varphi_1$ (appendix A).

Entering the ferromagnetic plate, two scalar solitons create a vector pulse. Since the z-component of the magnetization slowly vary in space around the pulses, the two pulse components mutually influence their propagation via contributing to the summary deviation of m_z from the equilibrium value M. The simultaneous excitation of the two ferromagnet regions shifted from each other by Δz along the z-direction is described using the pair of the Holstein–Primakoff transformations

$$\begin{aligned} m_{+1} &= (2M - 2a_2^* a_2)^{1/2} \left(1 - \frac{a_1^* a_1}{2M - 2a_2^* a_2}\right)^{1/2} a_1, \\ m_{+2} &= (2M - 2a_1^* a_1)^{1/2} \left(1 - \frac{a_2^* a_2}{2M - 2a_1^* a_1}\right)^{1/2} a_2, \\ m_{z1} &= m_{z2} = M - a_1^* a_1 - a_2^* a_2. \end{aligned} \tag{26}$$

From the pair of the Landau–Lifshitz equations, via the slowly varying envelope approximation, one finds the equations of motion for the envelopes of a_j in the form of the coupled nonlinear Schrödinger equations

$$\mathrm{i}\frac{\partial \tilde{a}_j}{\partial \tilde{t}} + \frac{\partial^2 \tilde{a}_j}{\partial \tilde{x}^2} + \gamma k^2 \left(\sum_{l=1}^{2} \tilde{a}_l^* \tilde{a}_l \right) \tilde{a}_j = 0. \tag{27}$$

The solution of (27)

$$\tilde{a}_j = \frac{c_j \sqrt{2}\zeta''}{\sqrt{\gamma k}} \,\mathrm{e}^{\mathrm{i}[\zeta' \tilde{x} + (\zeta''^2 - \zeta'^2)\tilde{t}]} \,\mathrm{sech}[\zeta''(\tilde{x} - \tilde{x}_0 - 2\zeta' \tilde{t})], \tag{28}$$

$\boldsymbol{c} = (c_1, c_2), |\boldsymbol{c}| = 1$, describes the propagation of the vector (Manakov) solitons [5]. Considering the scalar soliton as a spatially confined system of quantum particles (magnons) the number of which is conserved, one finds its width to be proportional to the magnon number $\zeta_j'' \propto n_j$, [11]. Since the magnons are not annihilated when incoming the plate and they are not exchanged between both the vector-pulse components, the ratio of the pulse-component intensities satisfies $|c_1|^2/|c_2|^2 = n_1/n_2 = \zeta_1''/\zeta_2''$. The last relation is found evaluating the intensities of the scalar solitons and vector-pulse components following the identities $\int_{-\infty}^{\infty}[\xi \,\mathrm{sech}(\xi x)]^2 \,\mathrm{d}x = 2\xi$, $\int_{-\infty}^{\infty} \left[\xi_j \,\mathrm{sech}\left(\sqrt{\xi_1^2 + \xi_2^2} x\right)\right]^2 \mathrm{d}x = 2\xi_j^2 \Big/ \sqrt{\xi_1^2 + \xi_2^2}$.

The vector-soliton stability against the energy exchange between different vector-pulse components demands lack of the diffraction of the main soliton mode following the condition $2\pi/|k_x| > d$ (the diffraction condition $2\pi/|k_x| = d\sin(\alpha)$ cannot be fulfilled for any α). Additionally, in order to prevent (to reduce) the exchange of short-wavelength modes (magnons) belonging to different vector-pulse components, one can confine them to the relevant plate area via a specific shape of the plate cross section shown in the inset of figure 1. Decreasing the ratio d_2/d_1 (see figure 1), one enhances the internal reflection of the soliton-component paths for the short-wavelength part of the pulse.

Under the condition $\zeta_j''^2 \ll \zeta'^2$, the constant phase factors $c_j/|c_j|$ of both the pulse components relate to the phase factors of the incoming scalar solitons following $c_j/|c_j| = \mathrm{e}^{\mathrm{i}\varphi_j}$ (envelopes (25) and (28) differ only by their widths $1/\zeta_j''$ and $1/\zeta''$, respectively). This is because the soliton conversion does not induce local change of the pulse shape, while it results in a homogeneous squeezing of whole the pulse. Thus, unlike in the case of the collision of solitons (propagating in the same wire) where a temporal local deformation of the colliding pulse (whose value depends on the width of the second pulse) is the reason of the phase and pulse-center shifts, the pulse conversion is not accompanied by shifts of the phases of the polarization components $c_j/|c_j|$ with relevance to φ_j. Let us note here that the collision of solitons of essentially different widths induces a small phase shift of the narrow (almost undeformed) pulse and a significant phase shift of the wide (locally deformed) pulse.

We conclude that the parameters of a pair of coherently propagating scalar solitons are identical to or directly relate (pulse widths) to the relevant parameters of the two-component vector soliton that is created from this pair; however, the vector pulse envelope is narrower than envelopes of the plate-incoming scalar pulses.

3.2. *Circuit of long Josephson junctions*

In the second soliton transmission line, two parallel long Josephson junctions connected via a common superconducting plate income two independent long Josephson junctions, (the superconducting plate is cut on some distance, see figure 2). The Manakov-like soliton (a vector fluxon) decays into two usual (scalar) fluxons incoming the area of independent long junctions.

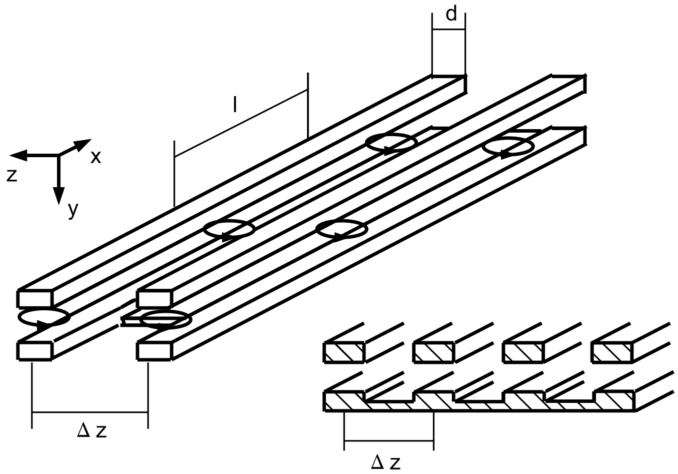


Figure 2. The double Josephson transmission line for conversion of pairs of (scalar) fluxons into vector soliton. Inset of the figure shows the cross section of the transmission line (superconducting plates) of the four-component fluxons.

We describe the simultaneous propagation of solitons (along the x-axis) in the two parallel long Josephson junctions separated (along the z-axis) by a distance Δz. The pair of junctions is built by attaching four superconducting plates to an insulator layer laid in the xz-plane, thus, the Josephson currents of both the junctions are directed along the y-axis. For each of the Josephson junctions, the Cooper-pair wavefunction is a superposition of the eigenstates relating to the occupation of the down ($|\mathcal{C}\rangle$) and up ($|\mathcal{D}\rangle$) superconducting plates

$$|\Psi(x,z,t)\rangle = \mathcal{D}(x,z,t)\,\mathrm{e}^{-\mathrm{i}\omega_{\mathcal{D}}t}|\mathcal{D}\rangle + \mathcal{C}(x,z,t)\,\mathrm{e}^{-\mathrm{i}\omega_{\mathcal{C}}t}|\mathcal{C}\rangle. \tag{29}$$

Tunneling through the insulator, the Cooper pair decays emitting a photon. The Hamiltonian of the system including the tunneling-induced photon field takes the form

$$\begin{aligned}\mathcal{H} = \omega_{\mathcal{D}}\mathcal{D}^*\mathcal{D} + \omega_{\mathcal{C}}\mathcal{C}^*\mathcal{C} + \frac{1}{\bar{c}^2}\frac{\partial A_y^*}{\partial t}\frac{\partial A_y}{\partial t}\\ + \frac{\partial A_y^*}{\partial x}\frac{\partial A_y}{\partial x} + \frac{\partial A_y^*}{\partial z}\frac{\partial A_y}{\partial z} + 2\pi\rho e\frac{c^2}{\bar{c}^2}[\mathrm{i}A_y(\mathcal{C}^*\,\mathrm{e}^{\mathrm{i}\omega_{\mathcal{C}}t})(\mathcal{D}\,\mathrm{e}^{-\mathrm{i}\omega_{\mathcal{D}}t}) + \mathrm{c.c.}],\end{aligned} \tag{30}$$

where A_y denotes the y-component of the vector potential of the electromagnetic field (the other components $A_x = A_z = 0$), ρ denotes the density of the superconductivity electrons, $\bar{c}$ is the Swihart velocity [12], $\bar{c} \ll c$. Defining new dynamical variables

$$E_y \equiv -\frac{1}{\bar{c}}\frac{\partial A_y}{\partial t}, \qquad P \equiv \mathrm{i}2(\mathcal{C}\,\mathrm{e}^{-\mathrm{i}\omega_{\mathcal{C}}t})(\mathcal{D}^*\,\mathrm{e}^{\mathrm{i}\omega_{\mathcal{D}}t})\,\mathrm{e}^{-\mathrm{i}\delta t}, \qquad D \equiv \mathcal{C}^*\mathcal{C} - \mathcal{D}^*\mathcal{D} \tag{31}$$

(here $\delta \equiv \omega_{\mathcal{D}} - \omega_{\mathcal{C}} + \bar{c}k$ denotes a photon-frequency detuning), using the slowly varying envelope approximation $A_y(x,z,t) = \tilde{A}(x,t)\cos(k_z z)\,\mathrm{e}^{\mathrm{i}(k_x x - \bar{c}kt)}$, $E_y(x,z,t) = \tilde{E}(x,t)\cos(k_z z)\,\mathrm{e}^{\mathrm{i}(k_x x - \bar{c}kt)}$, $P(x,z,t) = \tilde{P}(x,t)\cos(k_z z)\,\mathrm{e}^{\mathrm{i}(k_x x - \bar{c}kt)}$ and rescaling the electric-field envelope $\tilde{\mathcal{E}} \equiv (2\pi\rho e c^2/\bar{c}^2 k)\tilde{E}$, one arrives at the equations similar to those describing the self-induced transparency phenomenon in quantum optics

$$\frac{\partial\tilde{\mathcal{E}}}{\partial\tilde{x}} = -\alpha\tilde{P}, \qquad \frac{\partial\tilde{P}}{\partial\tilde{t}} = -2\tilde{\mathcal{E}}D, \qquad \frac{\partial D}{\partial\tilde{t}} = \tilde{\mathcal{E}}\tilde{P}^* + \tilde{\mathcal{E}}^*\tilde{P} \tag{32}$$

for $y = z = 0$. Here $\tilde{x} = x, \tilde{t} = t - x/\bar{c}$ and $\alpha \equiv (\pi \rho e c^2/\bar{c}^2)^2 2/(k + k_x)$. Searching for equations (32), we have used the relations

$$E_y \approx \mathrm{i}kA_y \approx \frac{k}{k + k_x}\left(\frac{\partial A_y}{\partial x} - \frac{1}{\bar{c}}\frac{\partial A_y}{\partial t}\right). \tag{33}$$

Below, the index of the dynamical variables ($j = 1, 2$) corresponds to one of the two Josephson junctions of a distance Δz from each other. The soliton solutions of (32) (see appendix B) take the form

$$\tilde{\mathcal{E}}_j = \zeta_j'' \,\mathrm{e}^{\mathrm{i}[\zeta'(\tilde{t} - 2\alpha\tau_j^2\tilde{x}) + \varphi_j]} \operatorname{sech}\left\{\zeta_j''\left[\tilde{t} + 2\alpha\tau_j^2(\tilde{x} - \tilde{x}_0)\right]\right\}. \tag{34}$$

For $\zeta_j''^2 \ll \zeta'^2, \tau_1 \approx \tau_2 \approx \sqrt{\zeta_1''^2 + \zeta_2''^2 + \zeta'^2} \equiv \tau$. The inter-wire soliton interaction has to be minimized for the similar reason as that relevant to the circuits transmitting the magnetic solitons (given below equation (25)).

In the circuit area where one of the superconducting plates of the double Josephson transmission line is common for the two neighboring Josephson junctions (figure 2), the fluxons propagating in the left-hand side and right-hand side junctions cannot be considered as independent solitons. There, the Cooper-pair state is a superposition

$$|\Psi(x, z, t)\rangle = \sum_{j=1}^{2} \mathcal{D}_j(x, z, t)\,\mathrm{e}^{-\mathrm{i}\omega_{\mathcal{D}}t}|\mathcal{D}_j\rangle + \mathcal{C}(x, z, t)\,\mathrm{e}^{-\mathrm{i}\omega_{\mathcal{C}}t}|\mathcal{C}\rangle \tag{35}$$

with an eigenstate $|\mathcal{C}\rangle$ relating to the occupation of the wide (common for the two junctions) superconducting plate. The two coherent fluxons incoming the double junction simultaneously create a vector soliton which is a solution of the dynamical equations

$$\frac{\partial \tilde{\mathcal{E}}_j}{\partial \tilde{x}} = -\alpha \tilde{P}_j, \qquad \frac{\partial \tilde{P}_j}{\partial \tilde{t}} = -2\tilde{\mathcal{E}}_j D, \qquad \frac{\partial D}{\partial \tilde{t}} = \sum_{j=1}^{2}(\tilde{\mathcal{E}}_j \tilde{P}_j^* + \tilde{\mathcal{E}}_j^* \tilde{P}_j). \tag{36}$$

Here $\tilde{P}_j$ denotes the amplitude of $P_j \equiv \mathrm{i}2(\mathcal{C}\mathrm{e}^{-\mathrm{i}\omega_{\mathcal{C}}t})(\mathcal{D}_j^* \,\mathrm{e}^{\mathrm{i}\omega_{\mathcal{D}}t})\,\mathrm{e}^{-\mathrm{i}\delta t}$, $\tilde{\mathcal{E}}_j$—the envelope of $E_{yj} \equiv -1/\bar{c}\partial A_{yj}/\partial t$, and $D \equiv \mathcal{C}^*\mathcal{C} - \sum_{j=1}^{2}\mathcal{D}_j^*\mathcal{D}_j$. The solution of (36) (appendix B)

$$\tilde{\mathcal{E}}_j = c_j \zeta'' \,\mathrm{e}^{\mathrm{i}\zeta'(\tilde{t} - 2\alpha\tau^2\tilde{x})} \operatorname{sech}\{\zeta''[\tilde{t} + 2\alpha\tau^2(\tilde{x} - \tilde{x}_0)]\} \tag{37}$$

describes the propagation of the vector soliton. Such a vector fluxon is similar to the self-induced transparency soliton in an optical medium consisting of identical three-level atoms (of a Λ- or V-configuration) [7, 13].

The condition of the vector-soliton stability against the energy exchange between different vector-pulse components is analogous to the one that is relevant for the system of the previous subsection, $2\pi/|k_x| > d$. The exchange of short-wavelength modes belonging to different vector-pulse components can be reduced via a specific shape of the superconducting-plate cross section shown in the inset of figure 2. This plate shape reduces an undesired Josephson current in the inter-path areas (in the z-direction).

Following the previous subsection arguments, we evaluate the relations between the scalar and vector pulse parameters; $|c_1|^2/|c_2|^2 = n_1/n_2 = \zeta_1''/\zeta_2'', c_j/|c_j| = \mathrm{e}^{\mathrm{i}\varphi_j}$.

4. Acceleration of collision-based gate operations

We describe the method of acceleration of the one-cebit and two-cebit gate operations with relevance to the elementary operations of the quantum-logic simulation scheme of section 2 utilizing the coherent dynamical changing of the soliton-polarization dimension (section 3).

4.1. Collisions of vector magnetic solitons and vector fluxons

The dynamical equations of the scalar- and vector-pulse envelope (32), (36) are applicable to the propagation of the single pulses or coherent soliton trains; however, they are irrelevant for describing the soliton collisions in general case. The parameters of these equations depend on a wave vector $\boldsymbol{k}$, while the colliding pulses (fluxons) are accompanied by the electromagnetic waves of different wavevectors. However, the ferromagnetic wires of the first of our proposals of the transmission lines can transmit solitons of the same wavevector and different velocities similarly as the nonlinear optical fibers which stabilize pulses carrying photons of a single frequency. Thus, unlike for the fluxon case, (23) and (27) can describe even multi-soliton envelopes. Since (32) and (36) do not describe multi-pulse evolution, the direct solution methods for the soliton equations (in particular, the Hirota method) are inapplicable for the description of the fluxon collisions. Instead, we use the method of the inverse-scattering transform which enables one to find the asymptotic forms of the two colliding pulses without knowledge of the space and time dependence of any two-pulse envelope. Let us note that the original description of the collision of the self-induced-transparency solitons includes the irrelevance of (32) to the incoherent multi-pulse dynamics via averaging the right-hand side of the first equation of (32) with a spectral distribution [14, 15].

For the system of two-coupled (via a common superconducting plate) long Josephson junctions, the basic equations for performing the inverse-scattering transform are the dynamical equations for the amplitudes of occupation of the Cooper-pair eigenstates

$$\begin{aligned} \mathrm{i}\frac{\partial(\mathcal{C}\,\mathrm{e}^{\mathrm{i}\delta t/2})}{\partial t} &= -\frac{\delta}{2}\mathcal{C}\,\mathrm{e}^{\mathrm{i}\delta t/2} + \sum_{j=1}^{2}\tilde{\mathcal{E}}_j\mathcal{D}_j\,\mathrm{e}^{-\mathrm{i}\delta t/2}, \\ \mathrm{i}\frac{\partial(\mathcal{D}_j\,\mathrm{e}^{-\mathrm{i}\delta t/2})}{\partial t} &= \frac{\delta}{2}\mathcal{D}_j\,\mathrm{e}^{-\mathrm{i}\delta t/2} + \tilde{\mathcal{E}}_j^*\mathcal{C}\,\mathrm{e}^{\mathrm{i}\delta t/2}. \end{aligned} \tag{38}$$

Knowing the one-pulse solutions of (36) and performing the analysis of the asymptotic solutions of (38), one finds the transformation of the polarization vector $\boldsymbol{c}$ due to the vector-fluxon collisions. Since equations (38) are similar to those relating to the inverse-scattering decomposition of the coupled nonlinear Schrödinger equations (27) [5], the collision-induced polarization transform is common for both the problems [7]. By analogy to the results of [5, 7], one finds the soliton-polarization vector of envelope (28) or (37) to transform during the collision with another Manakov soliton following (3) and (4). The same conclusion holds for the many-component vector solitons (the multi-component solitons of the coupled nonlinear Schrödinger equations and of the self-induced transparency [8, 16]).

4.2. Fast information-register switching

Let us consider a 2^n-component vector soliton that serves as n-cebit information register. After coherently disjoining it (following section 3) into a 2^{n-1} two-component vector solitons, the one-cebit operations can be performed via simultaneous collisions of these pulses with other two-component (switching) solitons. After the collisions, the pulses have to coherently join into another 2^n-component vector soliton whose polarization represents the switched-state vector. Performing the two-cebit gate operation (*CNOT*), one has to disjoin the register soliton into 2^{n-2} four-component vector pulses which can be simultaneously switched via collisions with other four-component solitons.

Let us identify the consecutive pairs of the polarization components with the consecutive pairs of the paths on the ferromagnetic (or superconducting) plate of the systems of section 3. Performing any one-cebit operation on the first of n cebits, one disconnects the 2^n-component

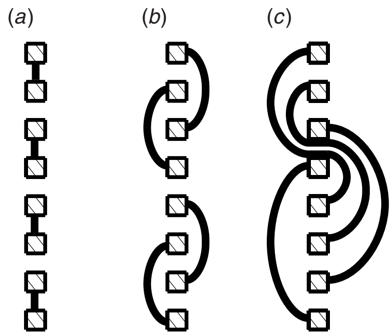


Figure 3. The cross sections of the magnetic circuit to perform one-cebit operations on: (*a*) the first cebit of the register, (*b*) the second cebit and (*c*) the third cebit.

vector soliton into two-component ones via cutting the ferromagnet (or superconducting) plate. Then, colliding the two-component pulses with the switching solitons one performs the transformation of the first cebit that is represented by a block-diagonal matrix (an example of such transformation is (17)). It is not enough to cut the plate in order to perform a one-cebit operation on another cebit. In particular, performing the *Hadamard* operation on the second cebit of the two-cebit register which is represented by (19), one has to disjoin all the soliton-component paths of the plate and to connect the first and third paths as well as the second and fourth paths by ferromagnetic (or superconducting) plates. In figure 3, connections necessary to perform one-cebit operations on a three-cebit register (whose state vector is an eight-component one) are visualized.

Unlike the previous proposal of the quantum-logic simulation via the vector-soliton collisions [3], the present one does not demand many consecutive collisions in order to perform an elementary gate operation. Since the collisions are performed simultaneously, the time consumed by this operation is independent of the number of the register cebits (while in the previous information-processing method, it scales exponentially with n). It is possible due to the nonlocality of the information carried by the magnetic (or Josephson-flux) vector pulses which enables the disconnection of the state-vector (polarization) components into observables related to spatially separated pulses. We note that the information carried by the previously considered vector solitons propagating in multi-component atomic condensates and media of multi-level atoms does not possess the property of nonlocality [4]. There is no way to disjoin such vector solitons into less-complex pulses since their polarization is relevant to an internal degree of freedom (spin) of the medium transmitting the solitons. The nonlocality in the present sense is different than that considered in [1] since information is not encoded in n physically separated cebits while it is encoded in 2^{n-1} separated objects (two-component solitons), thus, the coupling between the cebits cannot be controlled (one can only change the state-vector parameters). For this reason, the number of the pulse collisions relevant to a logical gate scales exponentially with n; however, the gate-operation time is independent of n.

4.3. Evolution of state coherence

Due to the macroscopic volume of the information carrier (the soliton), the decoherence is a continuous process which does not lead to an instant register-state damage. Since the state vector is defined at each time point and there is no sense of extending the computational state space in order to study the decoherence (analyzing the dynamics of the extended density matrix), there is no well-defined cebit-decoherence time. We show that, in terms of standard measures of the register-state coherence, the trace distance and fidelity [9], the present method of the information processing is not limited by the computation time. Using the time dependence of the fluctuations of the soliton-polarization parameters given in [4]; $\Delta\zeta_k''(t) = \Delta\zeta_{k0}''$, $\Delta\varphi_k(t) = \Delta\varphi_{k0} + \delta\varphi(t)$, where $\delta\varphi(t)$ is a linear function of t and depends on $\Delta\zeta_0'$, $\Delta\zeta_{l0}''$, $(l = 1, \ldots, 2^n)$, one finds the trace distance $1/2\sum_{k=1}^{2^n}|\Delta\zeta_k''(t)|/\zeta''$ and the fidelity $\sum_{k=1}^{2^n}[\zeta_k'' + \Delta\zeta_k''(t)]\zeta_k''/\zeta''^2$ to be independent of time. The velocity of the decoherence due to the evolution of $\Delta\varphi_k(t)$ is determined by the preparation of the initial register state (by values of $\Delta\zeta_0'$, $\Delta\zeta_{l0}''$), thus, there are only technological (not fundamental) limitations of its reducing.

We mention that in order to apply a method of the correction of the error induced by the evolution of $\Delta\varphi_k(t)$ that is described in [4], it would be useful to proceed with trains of the information-registering vector solitons instead with a single register pulse. Unlike for usual quantum information processing schemes (e.g. the linear-optics based one), the gate operations are not affected by the state readout, even in terms of the error correction. However, our method of conversion of the multi-component soliton into simpler pulses can be utilized in order to simplify the final-state readout (in particular, for the measurement of the phases φ_k) compared to the readout of the final state of the multi-component pulse considered in [4].

5. Discussion

We have described the possibility of performing quantum-logical operations with information encoded in mesoscopic objects, vector solitons, which makes the information stable against decoherence processes and easier to readout than the information encoded in quantum (microscopic) objects. Its realization demands the coherent dynamical changing the dimension of the soliton-polarization vector. Two soliton-transmission lines enabling such a vector-soliton conversion are proposed. These are the circuit of ferromagnetic wires transmitting the magnetic solitons and the circuit of long Josephson junctions transmitting the fluxons.

Several points to be discussed are crucial to the design of the soliton-circuit because of potential limitations to our information-processing method. First, one can be afraid if the discontinuity of the soliton equations at the points of change of the transmission-line cross-sections (the pulse-conversion points) can result in the spin-wave radiation (the magnetic circuit) or charge-density wave radiation (the fluxon circuit). Such radiation could influence the inter-pulse interaction (in particular, the interaction of the information-register with switching solitons and the interaction of the consecutive pulses of the register-soliton train) leading to the register-state decoherence [17]. However, the discontinuity is not enough reason for the radiation from the soliton. Its emission could appear due to a perturbation of the soliton equations. If the superposition of a linear-wave continuum and the soliton envelope satisfies the perturbed equations, the radiation from the soliton is emitted just after switching the perturbation on [18]. This situation is different from our one since the pulse equations have strict soliton solutions everywhere except the isolated points.

The second source of the register-state decoherence is the intrinsic dissipation of the soliton-transmission lines present in classic magnetic wires and long Josephson junctions. Dissipative terms of the soliton equations can be responsible for the radiation from solitons and they can influence the collision-induced polarization change of the vector solitons. According to [19], the dissipation (in particular, the one induced by structural imperfections of the Josephson junction) often strongly perturbs the equations of the fluxon motion. However, estimations have predicted that the fundamental need of the fluxon dephasing length to be bigger than its thickness (the Josephson penetration depth) could be fulfilled utilizing the transmission lines of the micrometer cross-section sizes [20]. Furthermore, novel possibilities of decreasing the transmission-line width and thickness using magnetic and superconducting nanowires, thus reducing the pulse volume [21, 22], give hope for further reducing the decoherence effects, although, there are fundamental limitations connected to the Mermin–Wagner–Hohenberg theorem on the phase-transition absence in one-dimensional systems. In particular, the magnetic nanowire technology has enabled (classical) logical operating via magnetic-soliton (domain-wall) colliding and cloning (we note that using the chains of magnetic quantum dots has also been explored) [23].

The third problem follows from the possibility of mutual shift of the pulse centers of the soliton ensemble. Since the switched two- or four-component solitons of the same velocity have to match into the 2^n-component vector pulse (the information register), they have to be centered at the same point of the x-axis. By the definitions given in section 2, all the switching pulses of the logical gates are of the property $|\zeta - \zeta_y^*|/|\zeta - \zeta_y| = 1$ ensuring lack of the collision-induced pulse-center shifting [5]. The mutual pulse-center shift can only be due to an intrinsic transmission-line noise and it can be minimized by shortening the distance of the independent propagation of the switched pulses.

Finally, let us mention that the coherent motion of complex fluxons is the problem investigated experimentally for other transmission-line configurations than proposed here, however [24]. Also, the fluxon propagation in multi-layer Josephson junctions, that is a similar transmission-line configuration to that enabling the vector-soliton propagation, was experimentally studied, however, for relatively weak inter-junction fluxon interactions [25]. The possibility of creating the vector magnetic solitons (in layered structures instead of in the continuous ferromagnet) was investigated only theoretically to the author's knowledge [26]. Studies of complex soliton transmission lines are expected to optimize the proposed method of the logical operating in terms of increasing the length of the coherent propagation of the soliton memory register and in terms of its state-readout efficiency.

Appendix A. Interaction of solitons of different parallel wires

We look for the conditions under which two scalar solitons of similar velocities that propagate in two parallel wires attract or repel each other. We assume that the transmission lines (the wires) are close to each other but they are not connected by any ferromagnetic (or superconducting) plate. Thus, the solitons of both wires can interact only via accompanying them with magnetic (or electromagnetic) field unlike solitons of the same wires which intra-wire interaction is due to their envelope overlapping.

We consider the solitary waves in the Heisenberg ferromagnet as the magnetostatic ones, thus, the magnetic field inside and outside the magnet satisfies the equations

$$\nabla \times \boldsymbol{h} = 0, \qquad \nabla \cdot (\boldsymbol{h} + 4\pi \boldsymbol{m}) = 0 \tag{A.1}$$

The first one of equations (A.1) suggests the existence of a scalar magnetic potential $\varphi(x, z, t)$, ($\boldsymbol{h} = \nabla\varphi$), whose space distribution can be found from the second equation of (A.1) [27]. Up to the leading order in k_x,

$$\frac{\partial^2 \varphi}{\partial z^2} = k_x^2 \varphi \tag{A.2}$$

for $|z| > d/2$, where d is the thickness of the wire centered along the x-axis, thus,

$$\varphi(x, z, t) = \varphi_0(x, t)\, \mathrm{e}^{\mathrm{i}(k_x x - \gamma M k^2 t)} \cos(k_z d/2)\, \mathrm{e}^{-k_x(|z| - d/2)} \tag{A.3}$$

in this area. Here $\varphi_0(x,t)$ is a slowly varying function. For $|z| < d/2$,

$$\varphi(x,z,t) = \varphi_0(x,t)\cos(k_z z)\,\mathrm{e}^{\mathrm{i}(k_x x - \gamma M k^2 t)} \tag{A.4}$$

since $\boldsymbol{h} \propto \nabla^2 \boldsymbol{m} \approx -k^2 \boldsymbol{m}$. The above form of $\varphi(x,z,t)$ ensures the continuity of h_x at the wire edges, at $|z| = d/2$, while h_z changes rapidly in the region near $|z| = d/2$ due to the magnetic ordering along the z-axis.

Studying the simultaneous soliton motion in two parallel wires, we index the dynamical variables relevant to the magnetic excitations in both wires by $j = 1, 2$ ($j = 1$ corresponds to a wire centered along the x-axis, $j = 2$ corresponds to a wire centered along the straight of $y = 0, z = \Delta z$). The main part of the energy of the interaction between the pulses takes the form

$$\begin{aligned} H_{\mathrm{int}} &= -\frac{1}{2}\int_{-\infty}^{\infty}\left[\int_{-d/2}^{d/2}\boldsymbol{m}_1(x,z,t)\boldsymbol{h}_2(x,z,t)\,\mathrm{d}z + \int_{-d/2+\Delta z}^{d/2+\Delta z}\boldsymbol{m}_2(x,z,t)\boldsymbol{h}_1(x,z,t)\,\mathrm{d}z\right]\mathrm{d}x \\ &\propto M k_x^2\,\mathrm{e}^{-k_x\Delta z}\int_{-\infty}^{\infty}[\tilde{a}_1^*(x,t)\tilde{a}_2(x,t) + \text{c.c.}]\,\mathrm{d}x. \end{aligned} \tag{A.5}$$

We insert the solutions of (23) shifted with respect to each other along the x-axis about a distance $\sqrt{\gamma M}(\tilde{x}_{01} - \tilde{x}_{02})$ (these are functions (25) with the parameters $\tilde{x}_{0j}$ taken instead of $\tilde{x}_0$) into (A.5). Modulus of the last integral in (A.5) decreases with $|\tilde{x}_{01} - \tilde{x}_{02}|$. In particular, for $\zeta_1'' = \zeta_2''$, one proves this fact from the identity $\int_{-\infty}^{\infty}\operatorname{sech}(x-a)\operatorname{sech}(x)\,\mathrm{d}x = 2a\cdot\operatorname{csch}(a)$ whose right-hand side decreases with $|a|$ in the whole range. However, its sign depends on the phase difference $\varphi_1 - \varphi_2$, thus, the interaction between the solitons of different wires can be repulsive or attractive. Similar dependence of the character of the soliton interaction (repulsion or attraction) on their phase difference is known to take place also between the pulses of a single wire [28].

Let us carry out the similar analysis of the fluxon interactions. Behind the area of the superconducting plate centered at $z = y = 0$, for $|z| > d/2$ (d denotes the width of the plate, the junction width), the non-zero component of the vector potential in the insulator layer (which matches two long Josephson junctions) satisfies the equation

$$\frac{1}{c^2}\frac{\partial^2 A_y}{\partial t^2} - \frac{\partial^2 A_y}{\partial x^2} - \frac{\partial^2 A_y}{\partial z^2} = 0. \tag{A.6}$$

For the single Josephson junction, its solution that fulfills the condition of the electric-field continuity is of the form

$$A_y(x,z,t) = \tilde{A}(x,t)\cos(k_z d/2)\,\mathrm{e}^{-\sqrt{k_x^2 - \bar{c}^2 k^2/c^2}(|z|-d/2)}\,\mathrm{e}^{\mathrm{i}(k_x x - \bar{c}kt)}. \tag{A.7}$$

For the simultaneous fluxon motion in two parallel junctions, the main part of the fluxon-interaction energy takes the form

$$\begin{aligned} H_{\mathrm{int}} &= \int_{-\infty}^{\infty}\left\{\left(\int_{-d/2}^{d/2} + \int_{-d/2+\Delta z}^{d/2+\Delta z}\right)\left[\frac{1}{\bar{c}^2}\frac{\partial A_{y1}^*}{\partial t}\frac{\partial A_{y2}}{\partial t} + \frac{\partial A_{y1}^*}{\partial x}\frac{\partial A_{y2}}{\partial x} + \frac{\partial A_{y1}^*}{\partial z}\frac{\partial A_{y2}}{\partial z} + \text{c.c.}\right]\mathrm{d}z\right\}\mathrm{d}x \\ &\propto \left(\frac{\bar{c}^2}{\pi\rho e c^2}\right)^2\left(k^2 + k_x^2\right)\mathrm{e}^{-\sqrt{k_x^2 - \bar{c}^2k^2/c^2}\Delta z}\int_{-\infty}^{\infty}[\tilde{\mathcal{E}}_1^*(x,t)\tilde{\mathcal{E}}_2(x,t) + \text{c.c.}]\,\mathrm{d}x. \end{aligned} \tag{A.8}$$

Here, the index of the dynamical variables ($j = 1, 2$) corresponds to one of the two Josephson junctions separated by a distance Δz from each other. Inserting the solutions of (32) shifted with respect to each other along the x-axis about a distance $|\tilde{x}_{02} - \tilde{x}_{01}|$ (these are functions (34) with the parameters $\tilde{x}_{0j}$ taken instead of $\tilde{x}_0$) into (A.8), one finds the inter-wire fluxon interaction to be repulsive or attractive depending on their phase difference in the same way as for the magnetic solitons propagating in the parallel wires. The dependence of the character of the fluxon interaction in a single Josephson junction on the phase difference is known and utilized in fluxonic devices [29].

Appendix B. One-soliton solution of multi-component fluxon equations

The one-pulse solution of (36) (function (37)) has been found using the bilinearization (Hirota) method. Combining equations (36) with the condition of normalization of the Cooper-pair wavefunction $\sum_{j=1}^{2}|P_j|^2 + D^2 = 1$, one arrives at

$$\frac{\partial^2\tilde{\mathcal{E}}_j}{\partial\tilde{t}\partial\tilde{x}} = 2\alpha\tilde{\mathcal{E}}_j\left(1 - \frac{1}{\alpha^2}\sum_{l=1}^{2}\left|\frac{\partial\tilde{\mathcal{E}}_l}{\partial\tilde{x}}\right|^2\right)^{1/2} \tag{B.1}$$

which is a vector version of the Getmanov equation [30]. Assuming $\tilde{\mathcal{E}}_j(\tilde{x},\tilde{t}) = 1/\tau g_j(\tilde{x},\tilde{t})/F(\tilde{x},\tilde{t})$, where $F(\tilde{x},\tilde{t})$ takes real values and $\tau = \text{const}$, one finds (B.1) to be satisfied if

$$\begin{aligned} &D_{\tilde{t}}D_{\tilde{x}}F\cdot F = 4\alpha\sum_{l=1}^{2}|g_l|^2,\\ &D_{\tilde{t}}D_{\tilde{x}}g_j\cdot F = 2\alpha g_j F,\\ &\sum_{j=1}^{2}|D_{\tilde{x}}g_j\cdot F|^2 = 4\alpha^2\tau^2\sum_{j=1}^{2}|g_j|^2\left(F^2 - \sum_{l=1}^{2}|g_l|^2\right). \end{aligned} \tag{B.2}$$

Here, $D_{\tilde{t}}$, $D_{\tilde{x}}$ denote Hirota operators of differentiation over the time and position, respectively, defined by

$$D_{\tilde{t}}^m D_{\tilde{x}}^n b(\tilde{x},\tilde{t})\cdot c(\tilde{x},\tilde{t}) \equiv (\partial/\partial\tilde{t} - \partial/\partial t')^m(\partial/\partial\tilde{x} - \partial/\partial x')^n b(\tilde{x},\tilde{t})c(x',t')|_{\tilde{x}=x',\tilde{t}=t'}. \tag{B.3}$$

Searching for equations (B.2), we have taken the additional condition $D = 1 - 2\sum_{j=1}^{2}|g_j|^2/F^2$ connected to the expectation that the form of the envelope components of the soliton reflects the occupation of the states $|\mathcal{D}_j\rangle$. Although the last equation of (B.2) is not a bilinear one, we look for its one-pulse solution in the form of the cut Hirota expansion $g_j = \alpha_j\,\mathrm{e}^{\eta}$, $F = 1 + \mathrm{e}^{\eta+\eta^*+R}$, with α_j, $R = \text{const}$ (similar as in the cases of trilinear decompositions of e.g. Landau–Lifshitz equation or Getmanov equation [30, 31]). One finds (B.2) to be satisfied for

$$\eta(\tilde{x},\tilde{t}) = \tilde{k}\tilde{t} + \frac{2\alpha}{\tilde{k}}\tilde{x} + \eta_0, \qquad |\tilde{k}|^2 = \frac{1}{\tau^2}, \qquad \mathrm{e}^R = \frac{1}{\tau^2(\tilde{k}+\tilde{k}^*)^2}. \tag{B.4}$$

In the main body of the text, we denote $\mathrm{Re}\,\tilde{k} \equiv \zeta''$, $\mathrm{Im}\,\tilde{k} \equiv \zeta'$.

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